Calculating Curvature Through Gradient Descent and Nonlinear Regression:

A Novel Mathematical Approach to Digital Anatomical Morphometry

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INTRODUCTION

Measurement of angular projection of structures has long been an established approach in anatomical morphometry, particularly in regards to bones. However, many of the described projection angles are in reference to inherently curved structures, often oversimplifying their topologies. Measuring the curvature of a projecting structure allows for a more accurate description of the structure's behavior in regards to its proximal-distal course in a given plane than simple angulations.

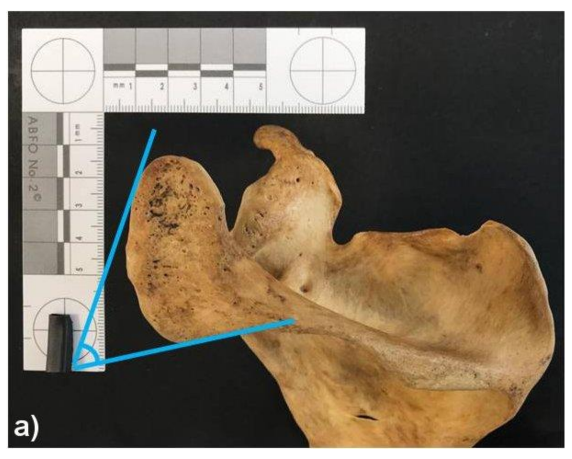
PURPOSE

To develop a quick, quantitative method for determining structural curvature from digital images.

MATERIALS & METHODS

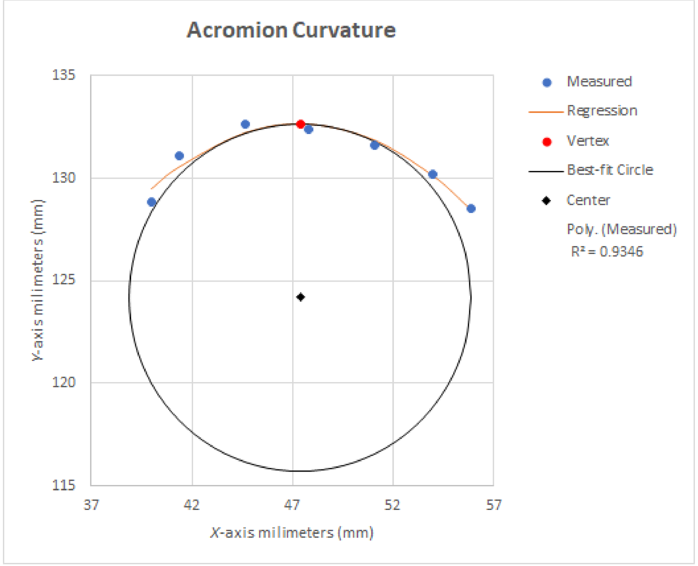
Experimental design:

- ➢Projection curvature was modeled on and assessed by the acromion from 50 dry scapulae and tested on 15 retrospective radiographs (AP projection) and 1 CT reconstruction.
- ➢Digital images were taken at a known scale off-axis to the axial plane.
- ➢Images were then processed in Fiji Image-J software where seven markers were placed along the interior and exterior curves of the acromion from base to apex.
- ➢The marker positions were recorded as pixel coordinates and imported into Excel.
- ➢Calculated curvature results were compared to angulation measurements in GraphPad Prism.



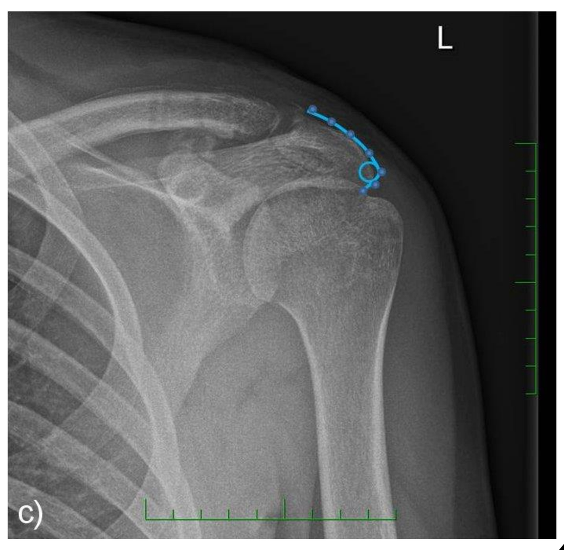
A. Angulation of the external acromion using a three-point measurement

B. Curvature of the external acromion using best-fit circle of a seven-point parabola.



Mathematical model design:

- ➢Utilizing Excel's *Solver* function (GRG Nonlinear, constraint precision = 10^{-6}, convergence = 10^{-9}, central derivative, Multistart, pop. size = 10^4, random seed = 0), the coordinate points were passed through a rotation matrix and optimized for second order regression. *Solver* was instructed to minimize sum of squared error by manipulating angle of point rotation and regression coefficients.
- ➢Outputted data reported acromion curvatures in mm^{-1} and R^2.

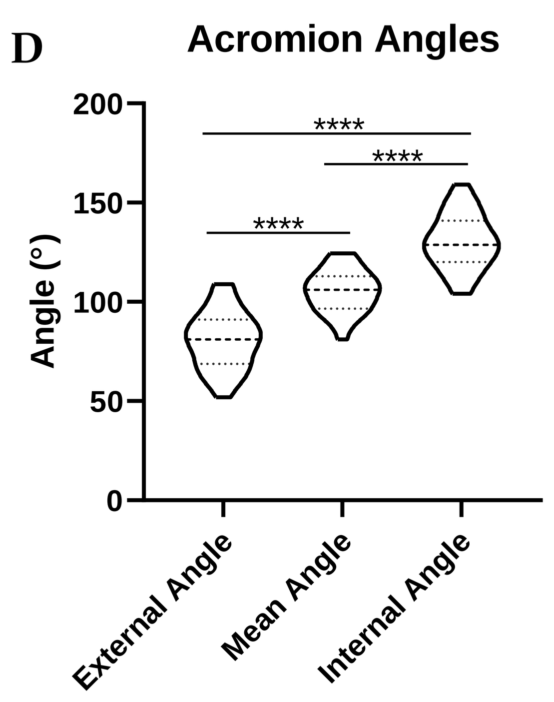


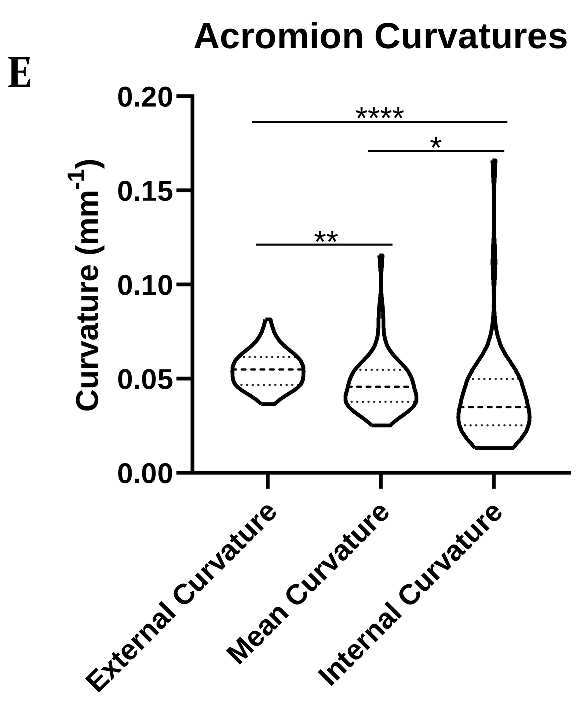
C. Testing the method on an X-ray sample.

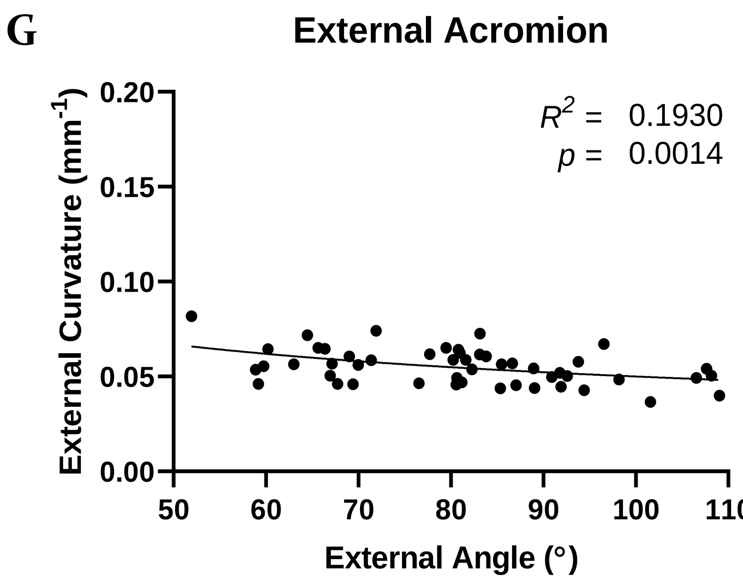
RESULTS & DISCUSSION

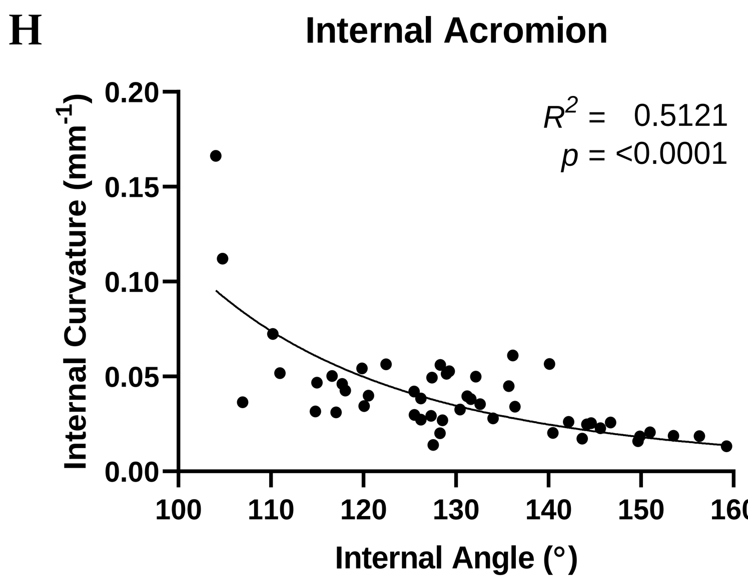
•Mean external acromion axial curvature was found to be 0.06 mm^{-1} ± 0.009 mm^{-1} at $\overline{R}^2$ = 0.99 ± 0.008.

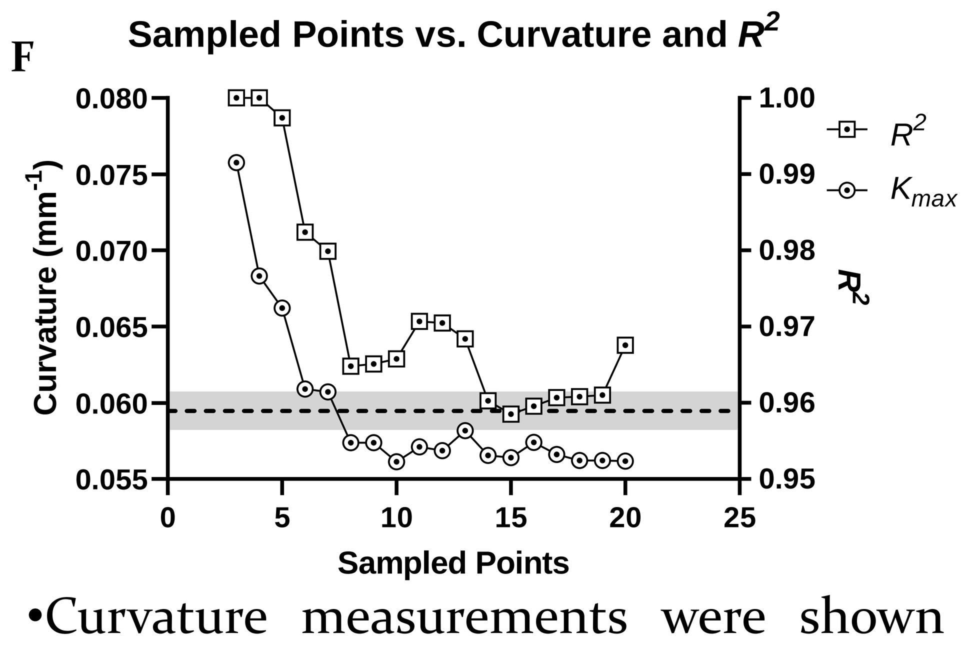
•Mean internal acromion axial curvature was found to be 0.04 mm^{-1} ± 0.025 mm^{-1} at $\overline{R}^2$ = 0.98 ± 0.036.









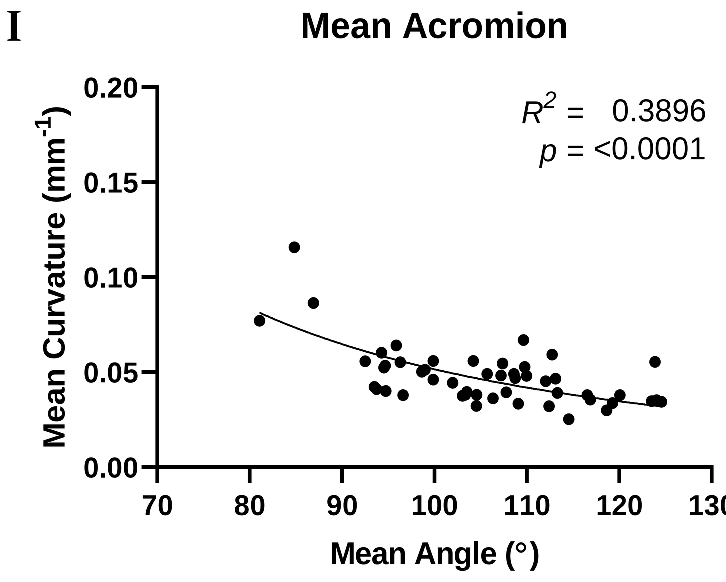


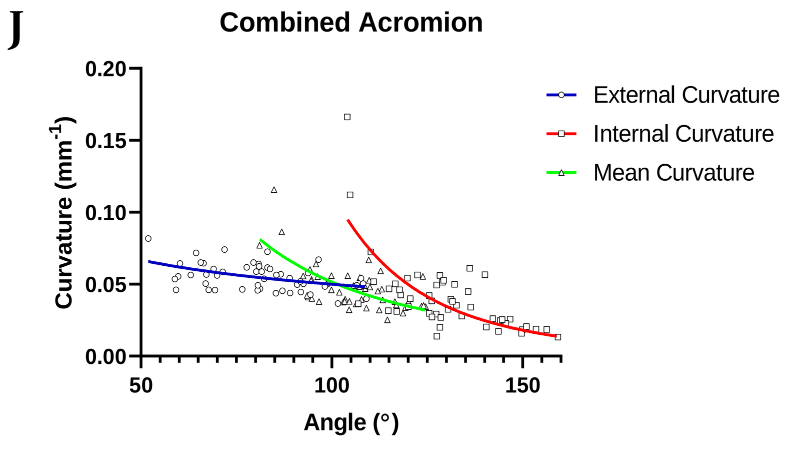
D: Ordinary one-way ANOVA between external, mean, and internal acromion angles.

E: Kruskal—Wallis one-way ANOVA between external, mean, and internal acromion curvatures.

F: Effect of number of sample points on curvature and R^2.

G—J: Linear regressions of acromion curvatures (calculated in lognormal space).





•Curvature measurements were shown to have a highly significant negative correlation to angle measurements, which demonstrates the interchangeability of the separate values. However, curvature is a more accurate and intuitive way to measure anatomical processes,.

•While mean acromion curvature showed significant nonzero bias, this is a calculated value and significant nonzero bias was also reported when comparing purely internal acromion angulation, which may indicate a greater degree of variability and observer ambiguity when measuring this projection. No bias was found when calculating internal acromion curvature between two observers.

•One-way ANOVAs showed significant differences ($p<0.05$) between measured groups.

•Linear regression showed trends between curvature and angulation, with significant negative correlation between the two measurements ($p<0.001$).

•Log transformations and non-parametric tests were employed in the case of non-Gaussian residuals in data sets..

CONCLUSION

Axial acromion measurements of curvature showed distinct categorizations between external, internal, and calculated mean curvatures. X-ray and CT radiographs were successful in proof of concept making this method candidate radiographical analyses in clinical settings.

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